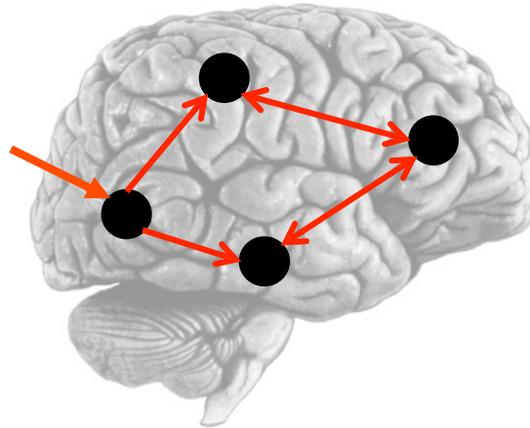


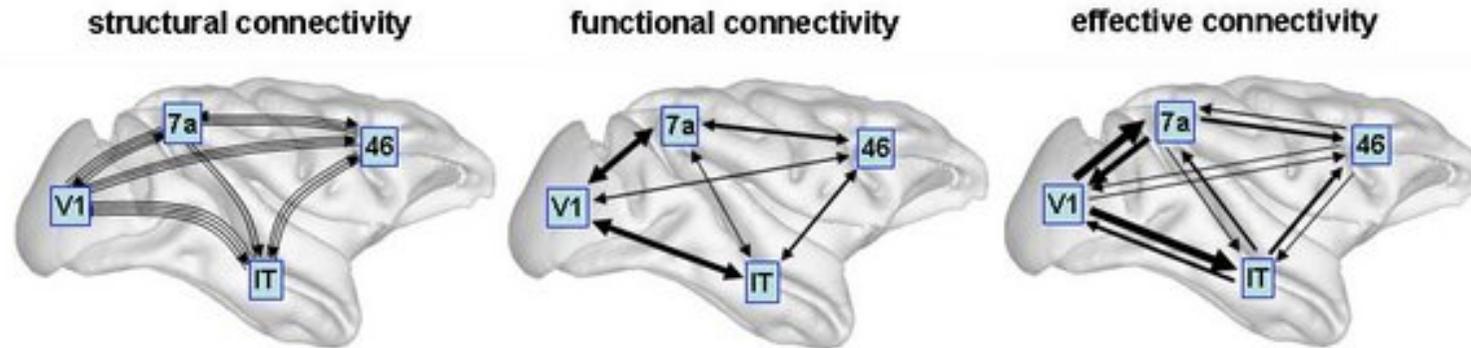
# Dynamic Causal Modelling: Tutorial



Rosalyn Moran, Wellcome Trust Centre for Neuroimaging  
University College London

MPS-UCL Symposium on Computational Psychiatry  
September 16 – 22, 2012, Ringberg Castle

# Structural, functional & effective connectivity



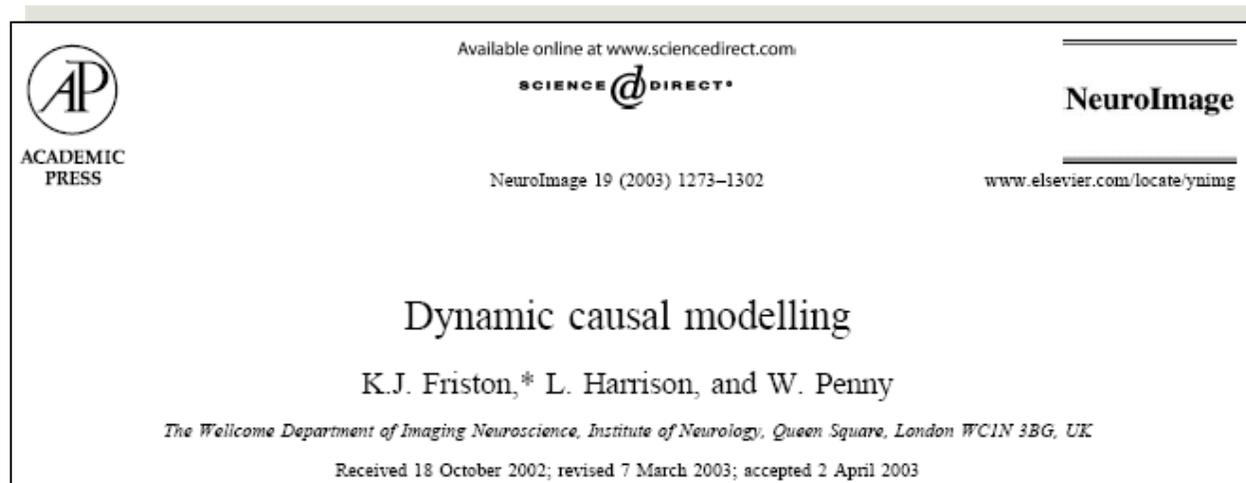
Sporns 2007, *Scholarpedia*

- **anatomical/structural connectivity**  
= presence of axonal connections
- **functional connectivity**  
= statistical dependencies between regional time series
- **effective connectivity**  
= causal (directed) influences between neurons or neuronal populations

# Methods for effective connectivity analysis

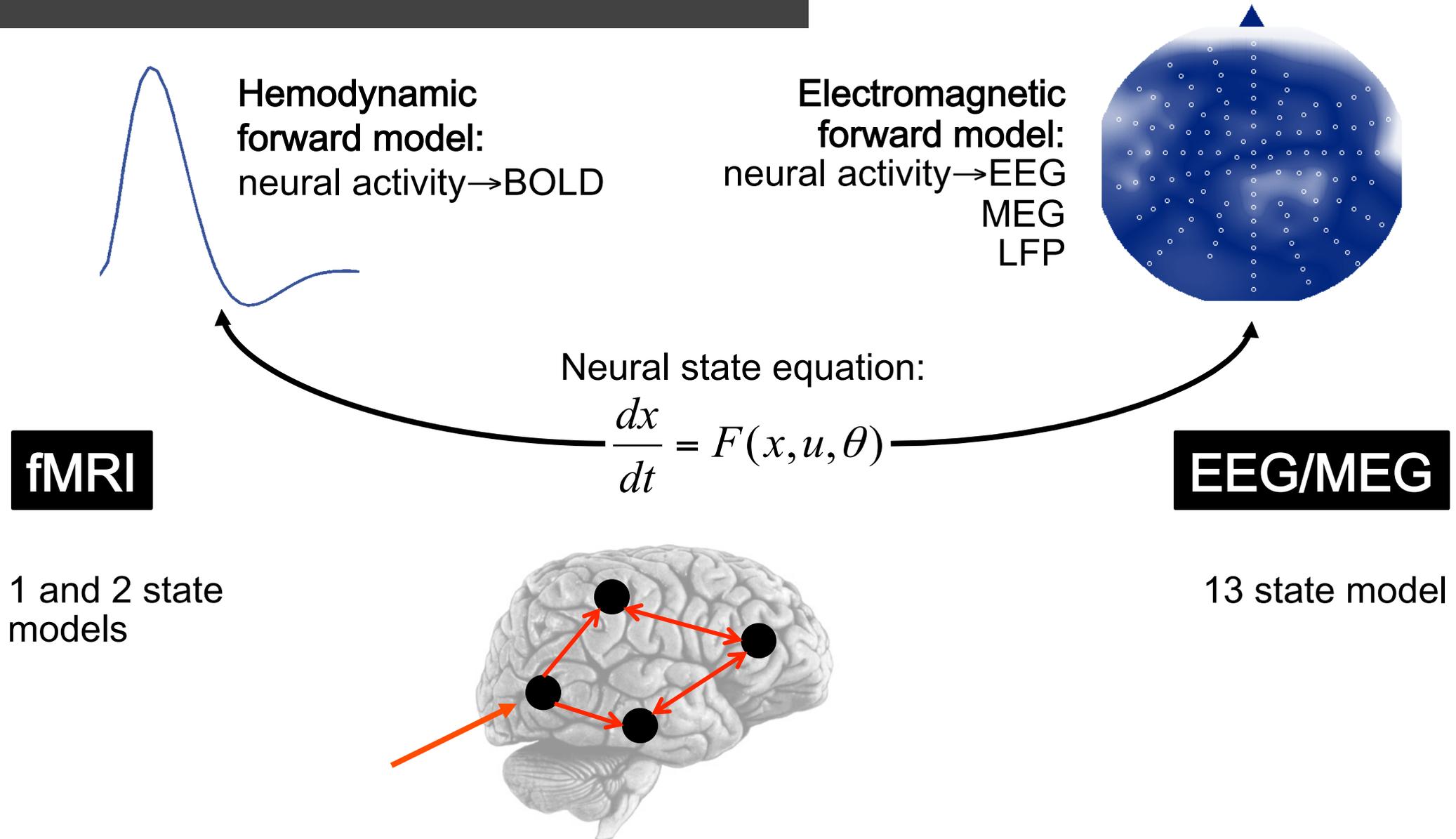
- Regression models  
(e.g. psycho-physiological interactions, PPIs)  
Friston et al. 1997
- Structural Equation Models (SEM)  
McIntosh et al. 1991, 1994; Büchel & Friston 1997; Bullmore et al. 2000
- Volterra kernels  
Friston & Büchel 2000
- Time series models (e.g. MAR, Granger causality)  
Harrison et al. 2003, Goebel et al. 2003
- Dynamic Causal Models (DCM)  
*bilinear*: Friston et al. 2003; *nonlinear*: Stephan et al. 2008; *ERPs* David et al 2006; *LFPs*, Moran et al 2009

# Dynamic causal modelling (DCM)



DCMs are generative models of brain responses, which provide posterior estimates of neurobiologically interpretable parameters such as the effective strength of connections among neuronal populations and their context dependent modulation

# A Generic Framework



# Its Key Components

## Dynamic:

Dynamic (differential) equations describe hidden neuronal dynamics at a level of detail constrained by the measurement

## Causal:

In a control theory sense, input perturbations disturb equilibrium neuronal dynamics and propagate through connected networks to other brain regions

## Biophysical Observer:

Realistic neuronal to measurement model accounts for regional HRF differences and captures key electrophysiological features eg. Power spectra

## Bayesian Inversion:

Priors on biological parameters to constrain them within physiologically plausible ranges  
Model evidence objective function maximised during parameter estimation  
Mean and covariance estimates for a parameter set

# Overview

- Dynamic causal models for fMRI
  - Work through example: Attention to Motion
  - Neural level & Hemodynamic level
  - Parameter estimation, priors & inference
- Dynamic causal models for Steady State Responses (rat LFPs)
  - Work through example: Isoflurane effects on connectivity
  - Neural mass model

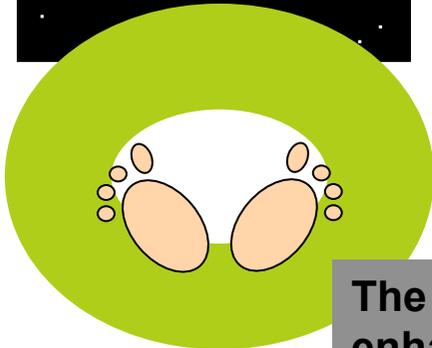
# Attention to Motion

## Paradigm



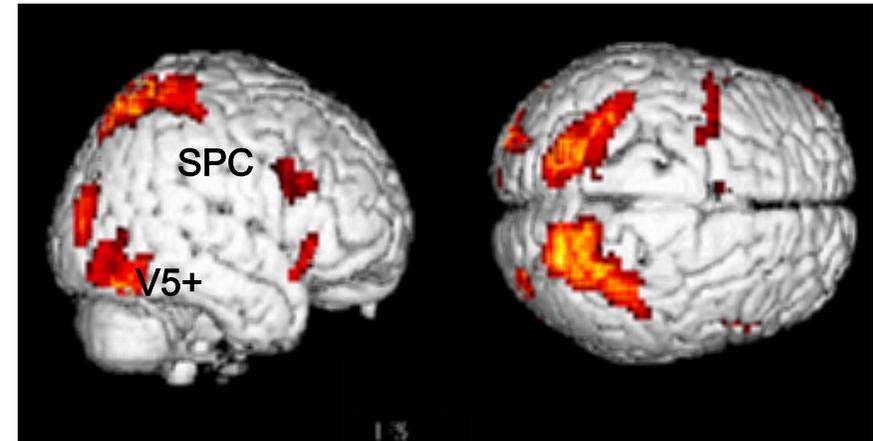
4 conditions

- fixation only
- observe static dots
- observe moving dots
- attend to moving dots



## GLM Results

Attention – No attention

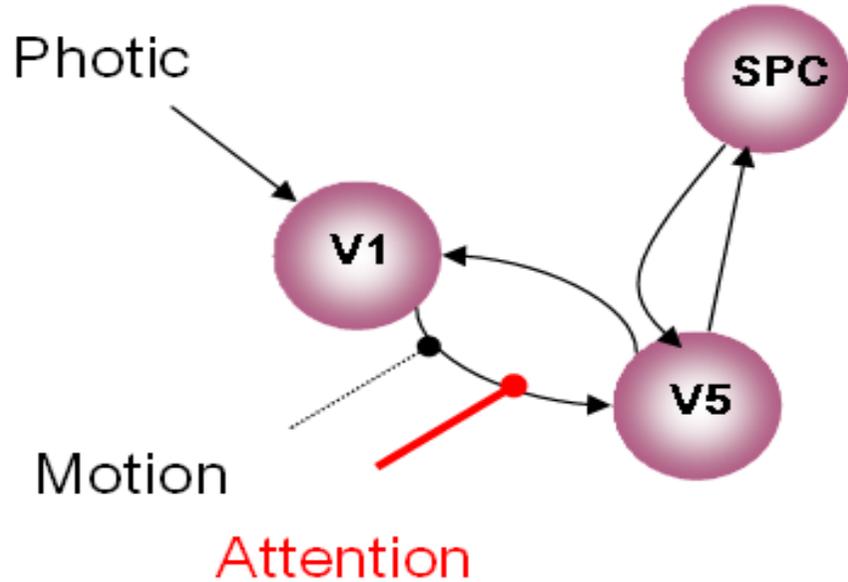


Büchel & Friston 1997, Cereb. Cortex  
Büchel et al. 1998, Brain

The GLM analysis showed that activity in area V5 was not only enhanced by moving stimuli, but also by attention to motion.

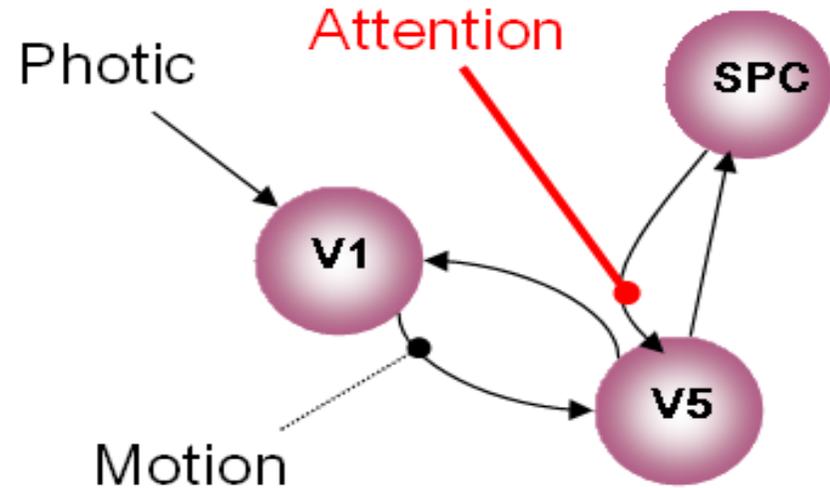
In the following we will model this effect in V5 using DCM, testing competing hypotheses regarding context-dependent modulation or “enabling” of V5 afferents.

# Effects of attention on motion responses in V5



**Model 1/Hypothesis 1  
Bottom-up Model**

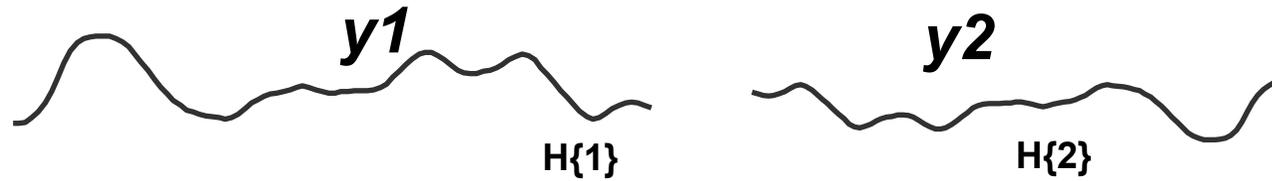
**Model 2/ Hypothesis 2  
Top-down Model**



# Overview

- Dynamic causal models for fMRI
  - Work through example: Attention to Motion
  - Neural level & Hemodynamic level
  - Parameter estimation, priors & inference
  
- Dynamic causal models for Steady State Responses (rat LFPs)
  - Work through example: Isoflurane effects on connectivity
  - Neural mass model

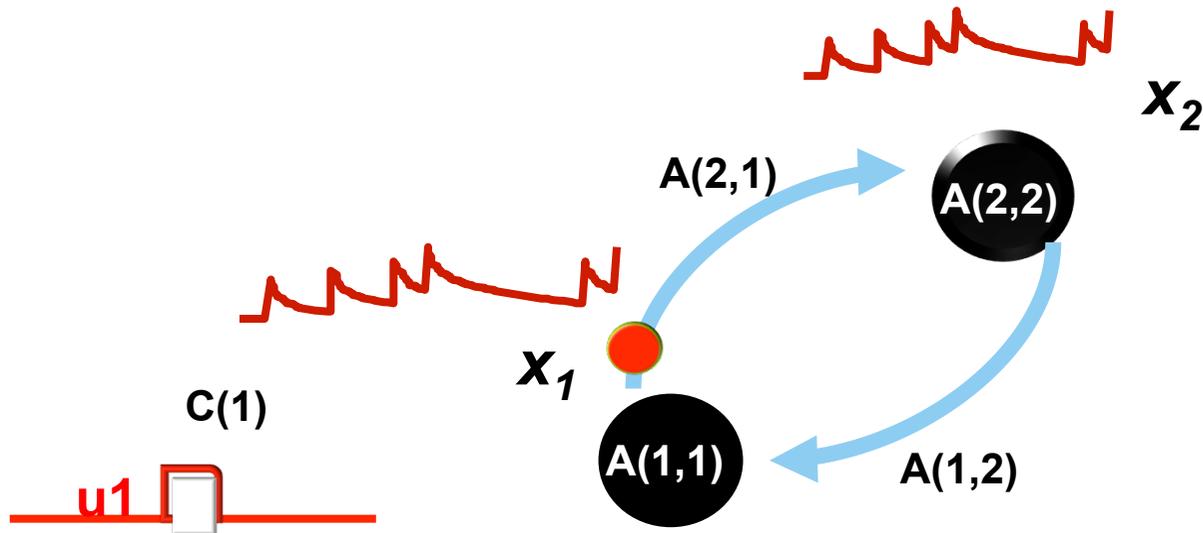
# Neural Dynamics and Static Observer



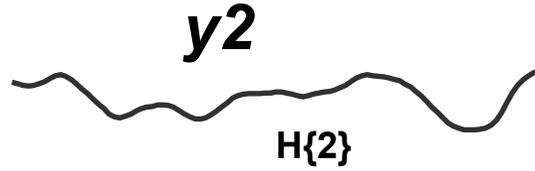
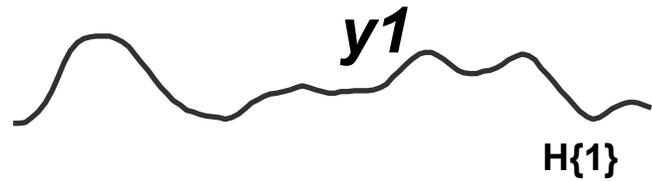
$$\dot{x} = Ax + Cu$$

$$y = g(x, H) + \varepsilon$$

$$\varepsilon \sim N(0, \sigma)$$



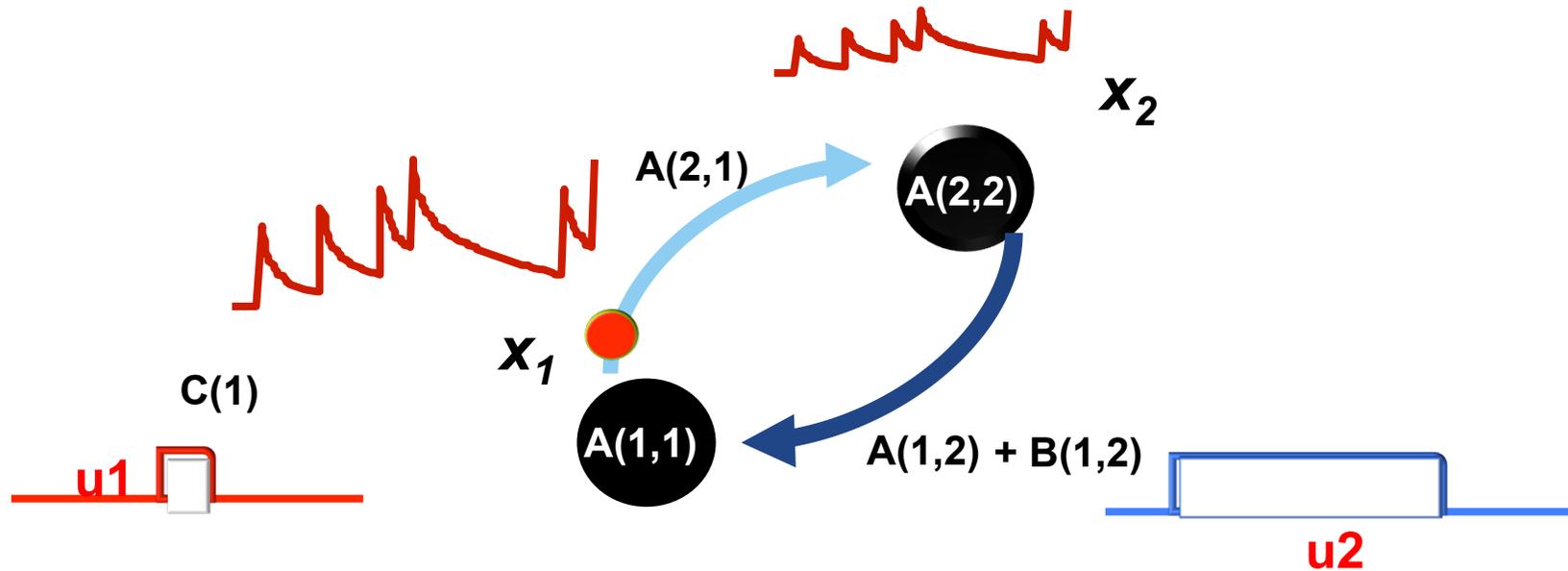
# Neural Dynamics and Static Observer



$$\dot{x} = (A + uB)x + Cu$$

$$y = g(x, H) + \varepsilon$$

$$\varepsilon \sim N(0, \sigma)$$



# Vanilla DCM: Neural Dynamics : a bilinear approximation

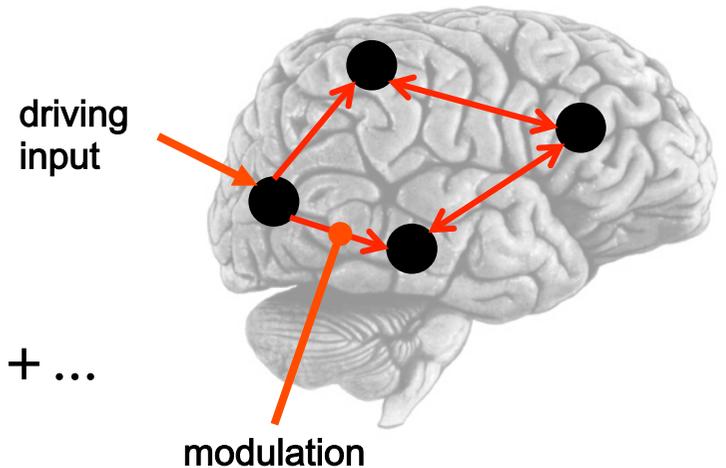
Simply a two-dimensional  
Taylor expansion (around  $x_0=0, u_0=0$ ):

$$\frac{dx}{dt} = f(x, u) \approx f(x_0, 0) + \frac{\partial f}{\partial x} x + \frac{\partial f}{\partial u} u + \frac{\partial^2 f}{\partial x \partial u} ux + \dots$$

$$A = \left. \frac{\partial f}{\partial x} \right|_{u=0}$$

$$C = \left. \frac{\partial f}{\partial u} \right|_{x=0}$$

$$B = \frac{\partial^2 f}{\partial x \partial u}$$



Bilinear state equation:

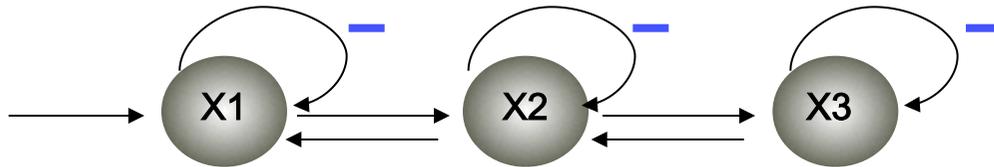
$$\frac{dx}{dt} = \left( A + \sum_{i=1}^m u_i B^{(i)} \right) x + Cu$$

# Connectivity Parameters: Rate Constants

Integration of a first-order linear differential equation gives an exponential function:

$$\frac{dx}{dt} = ax \longrightarrow x(t) = x_0 \exp(at)$$

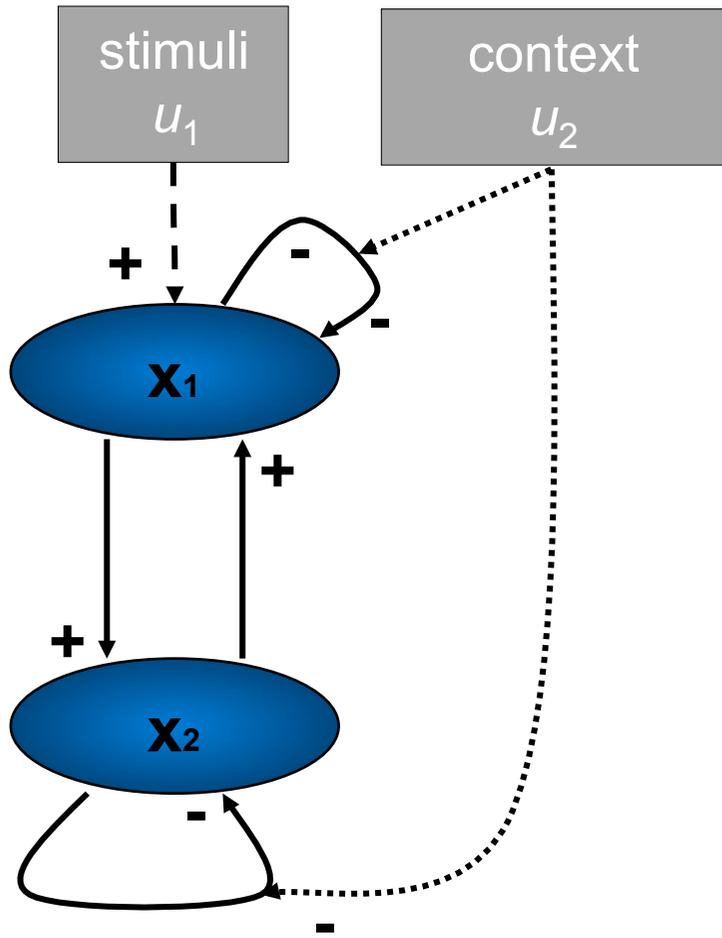
The coupling parameter  $a$  thus describes the speed of the exponential change



If  $A \rightarrow B$  is  $0.10 \text{ s}^{-1}$  this means that, per unit time, the increase in activity in B corresponds to 10% of the activity in A

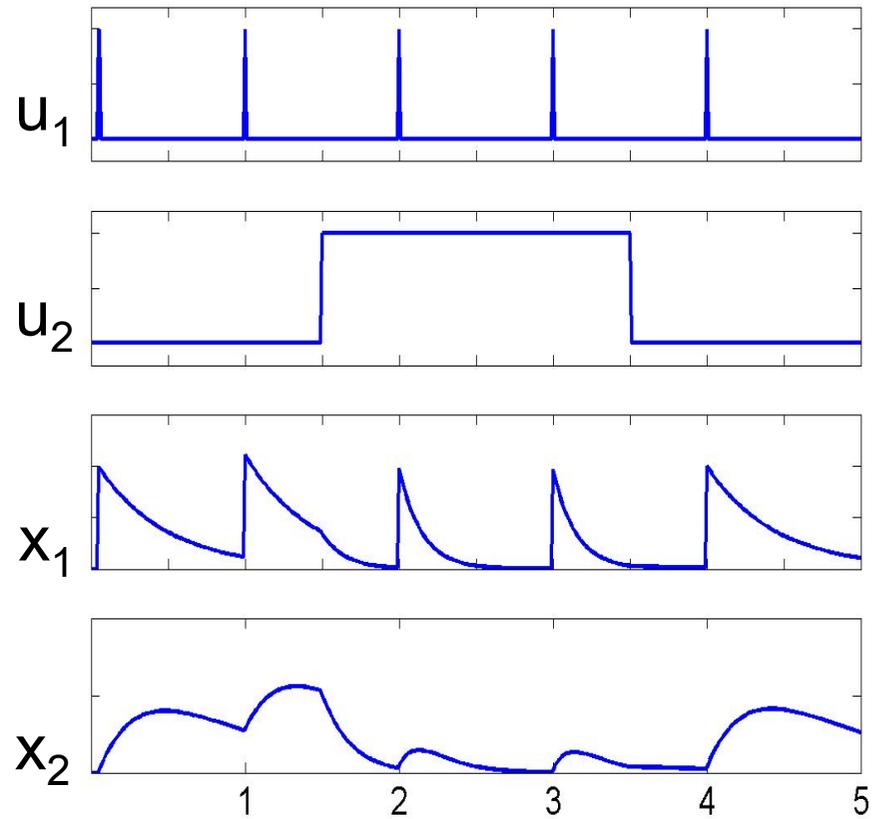
If  $A \rightarrow B$  is  $-0.10 \text{ s}^{-1}$  this means that, per unit time, the decrease in activity in B corresponds to 10% of the activity in A

# Context Dependent Decay



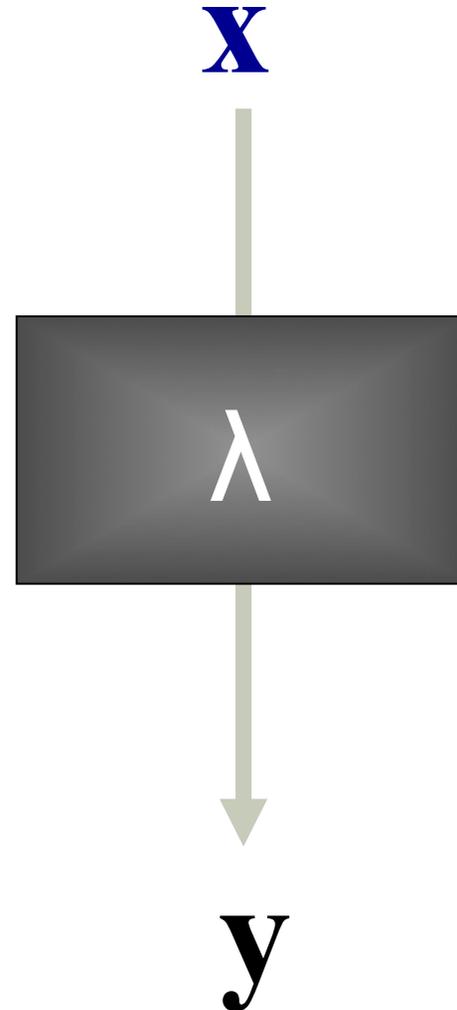
$$\dot{x} = Ax + u_2 B^{(2)} x + Cu_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & a_{12} \\ a_{21} & -1 \end{bmatrix} x + u_2 \begin{bmatrix} b_{11}^2 & 0 \\ 0 & b_{22}^2 \end{bmatrix} x + \begin{bmatrix} c_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$



# Neuronal activity to BOLD

- Cognitive system is modelled at its underlying neuronal level (not directly accessible for fMRI).
- The modelled neuronal dynamics ( $X$ ) are transformed into area-specific BOLD signals ( $y$ ) by a hemodynamic model ( $\lambda$ ).



# Hemodynamic Model

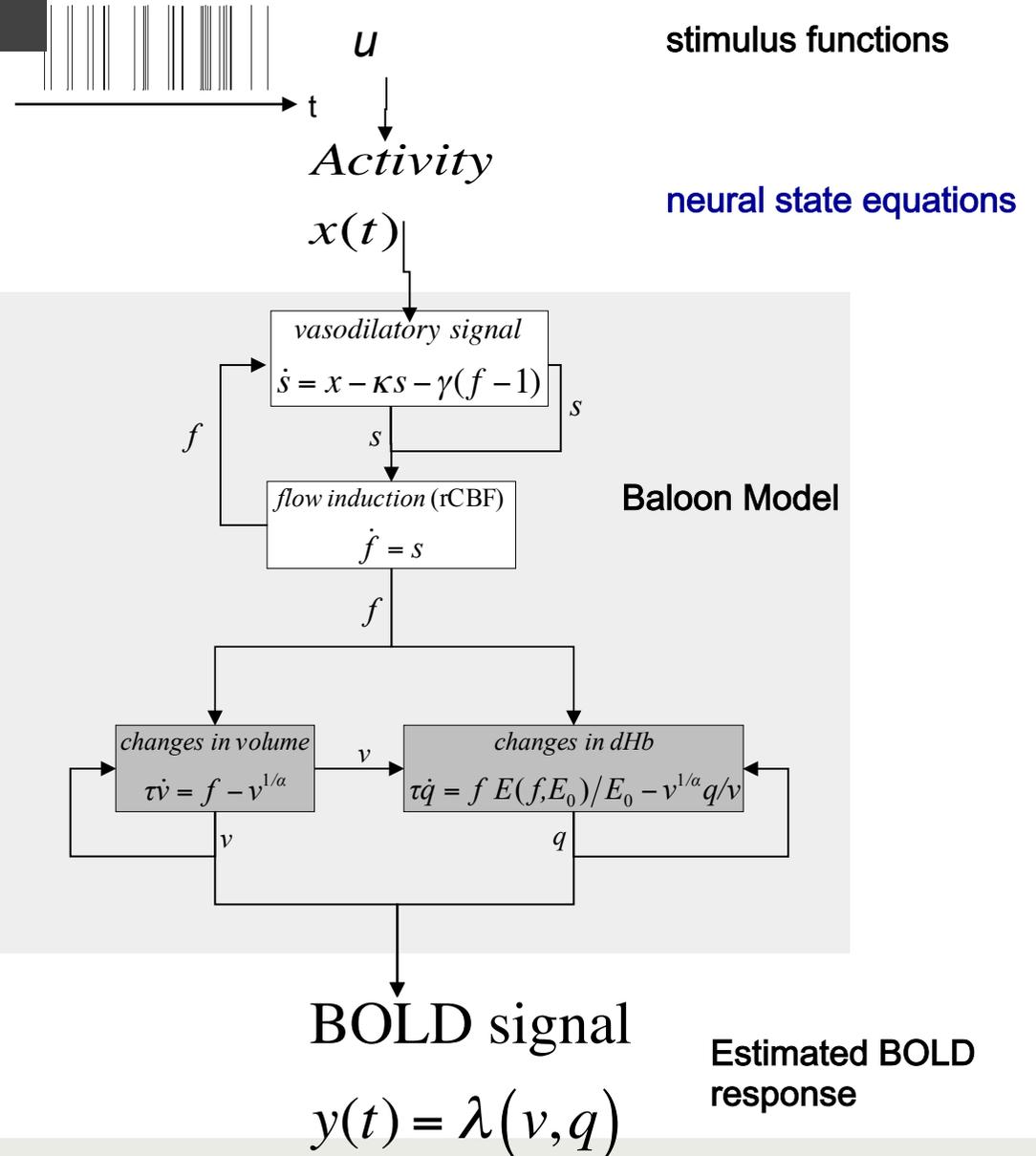
- 6 parameters:

$$\theta^h = \{K, \gamma, \tau, \alpha, \rho, \varepsilon\}$$

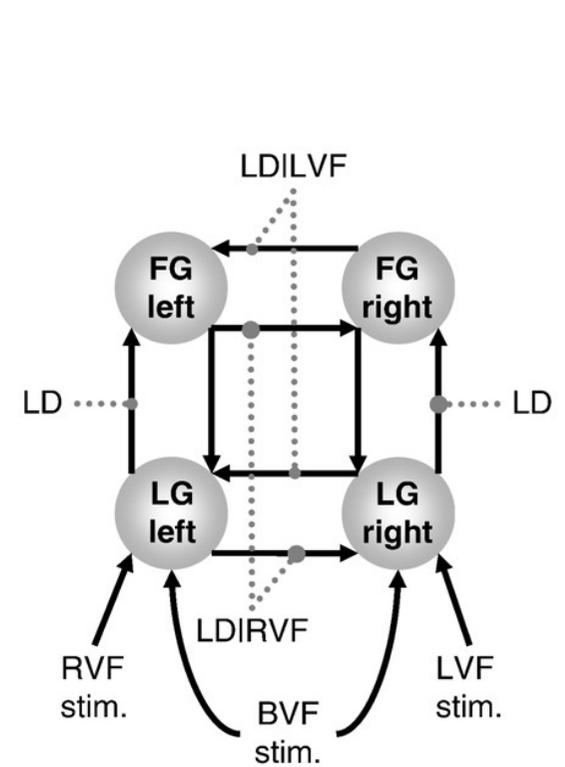
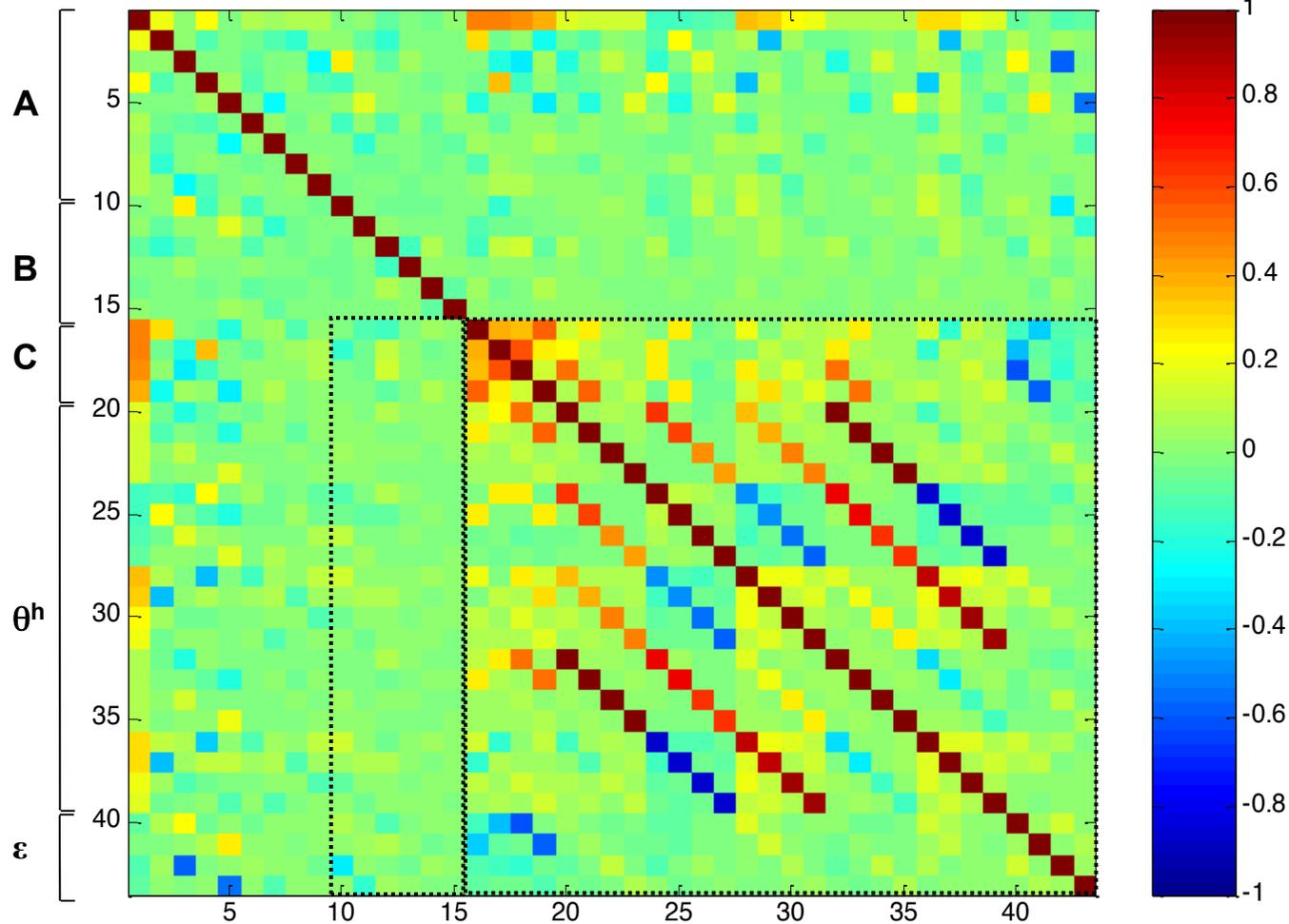


important for model fitting, but usually of no interest for statistical inference

- Computed separately for each area → region-specific HRFs



# Hemodynamic and Neural Parameter Correlations



# Overview

- Dynamic causal models for fMRI
  - Work through example: Attention to Motion
  - Neural level & Hemodynamic level
  - Parameter estimation, priors & inference
  
- Dynamic causal models for Steady State Responses (rat LFPs)
  - Work through example: Isoflurane effects on connectivity
  - Neural mass model

# Inference on Models

**Model evidence:**  $p(y | m_i)$

**Approximation: Free Energy**  $F = \ln p(y | m_i) - KL[q(\theta), p(\theta | G, \lambda)]$



accounts for both accuracy and complexity of the model



allows for inference about structure (generalisability) of the model

Fixed Effects Model selection via

log Group Bayes factor:

$$BF_{1,2} = \sum_k \ln p(y|m_1) - \sum_k \ln p(y|m_2)$$

Random Effects Model selection

via Model probability:

$$p(r | y, \alpha)$$

$$\langle r_k \rangle_q = \alpha_k / (\alpha_1 + \dots + \alpha_K)$$

# Inference on Models

**Model evidence:**  $p(y | m_i)$

**Approximation: Free Energy**  $F = \ln p(y | m_i) - KL[q(\theta), p(\theta | G, \lambda)]$

balance between fit and complexity = accuracy - *complexity*

$$F = \langle \log p(y | \theta, m) \rangle_q - KL[q(\theta), p(\theta | m)]$$

Independent Priors

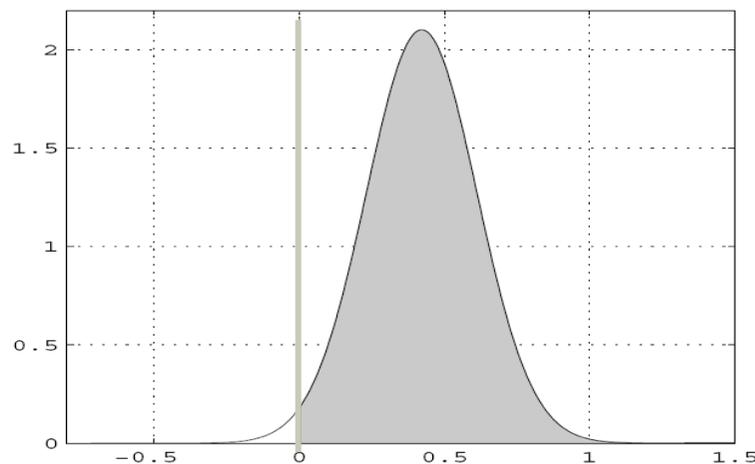
Deviation of posterior mean from prior mean

$$KL_{Laplace} = \frac{1}{2} \ln |C_\theta| - \frac{1}{2} \ln |C_{\theta|y}| + \frac{1}{2} (\mu_{\theta|y} - \mu_\theta)^T C_\theta^{-1} (\mu_{\theta|y} - \mu_\theta)$$

Dependent Posteriors

# Inference on Single Subject Parameters

- Gaussian assumptions about the posterior distributions of the parameters
- posterior probability that a certain parameter (or contrast of parameters) is above a chosen threshold  $\gamma$ :
- By default,  $\gamma$  is chosen as zero – the prior ("does the effect exist?").



# Inference on Multi Subject Parameters: FFX Bayesian Parameter Averaging

Likelihood distributions from different subjects are independent

Under Gaussian assumptions this is easy to compute:

group posterior covariance

individual posterior covariances

$$C_{\theta|y_1, \dots, y_N}^{-1} = \sum_{i=1}^N C_{\theta|y_i}^{-1}$$
$$\eta_{\theta|y_1, \dots, y_N} = \left( \sum_{i=1}^N C_{\theta|y_i}^{-1} \eta_{\theta|y_i} \right) C_{\theta|y_1, \dots, y_N}$$

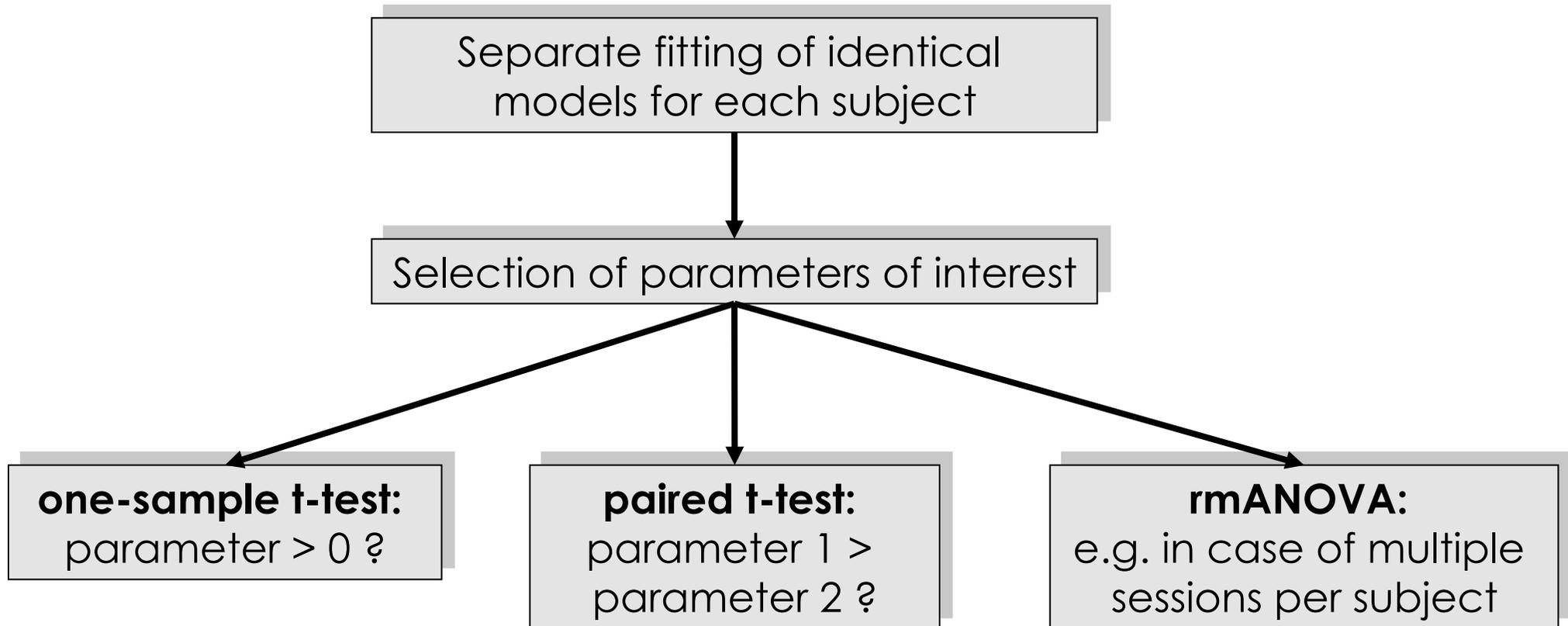
group posterior mean

individual posterior covariances and means

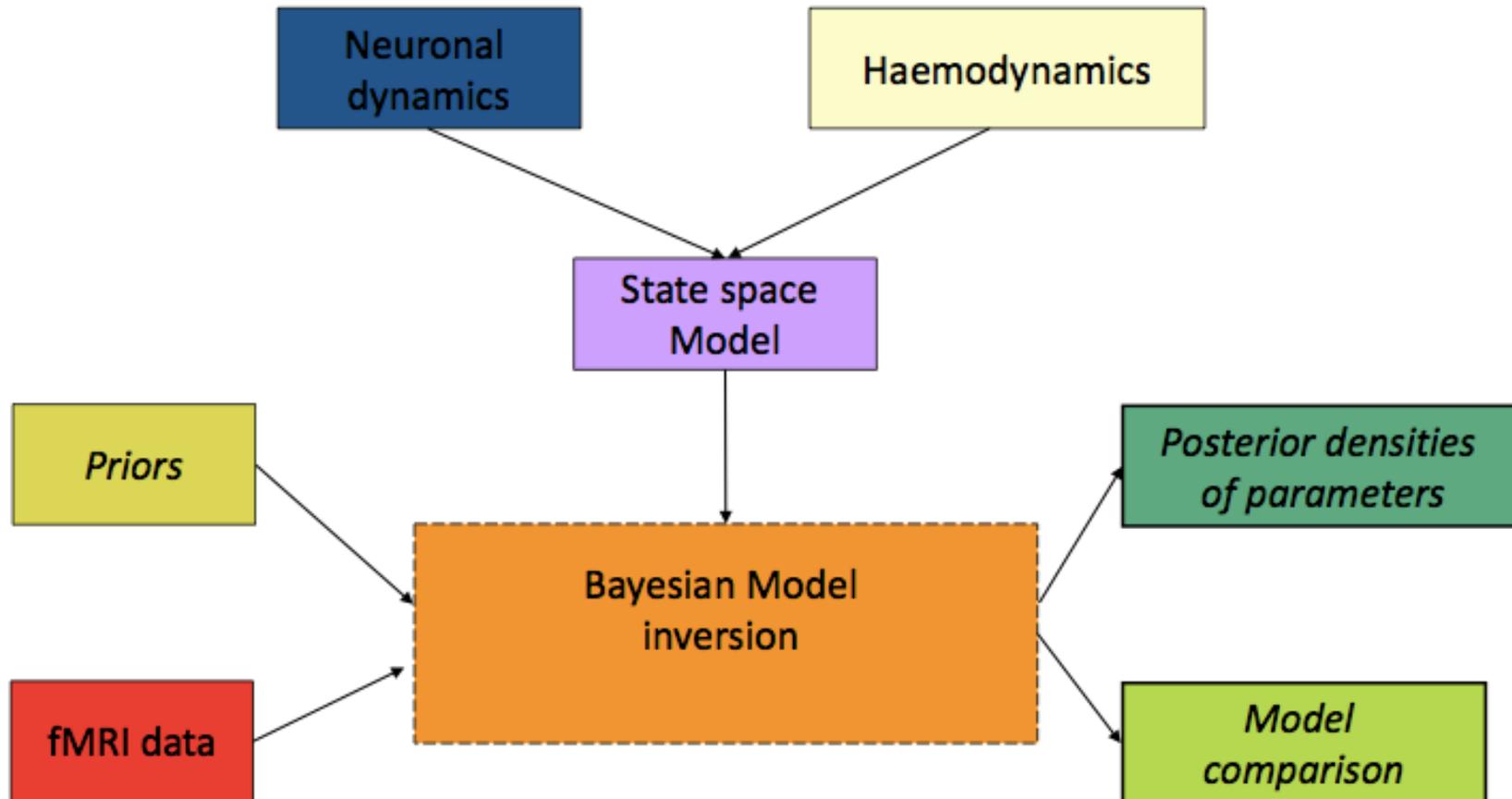
The diagram illustrates the process of Bayesian parameter averaging. It shows two equations within a box. The first equation relates the inverse of the group posterior covariance to the sum of the inverse of individual posterior covariances. The second equation relates the group posterior mean to a weighted sum of individual posterior means, where the weights are the inverse of individual posterior covariances. Arrows indicate the mapping from text labels to the mathematical terms: 'group posterior covariance' points to the left side of the first equation; 'individual posterior covariances' points to the sum in the first equation; 'group posterior mean' points to the left side of the second equation; and 'individual posterior covariances and means' points to the sum in the second equation.

# Inference on Multi Subject Parameters: RFX Summary Statistic Approach

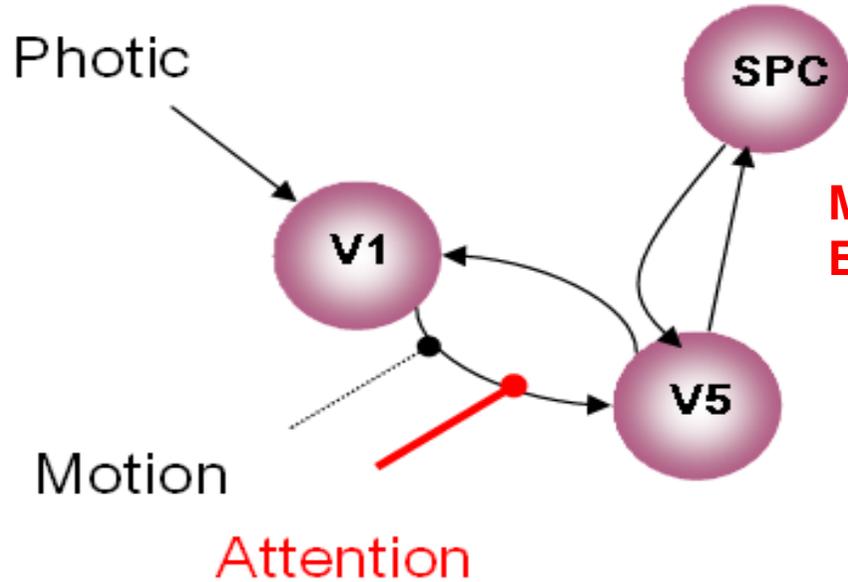
- In analogy to “random effects” analyses in SPM, 2<sup>nd</sup> level analyses can be applied to DCM parameters:



# Roadmap

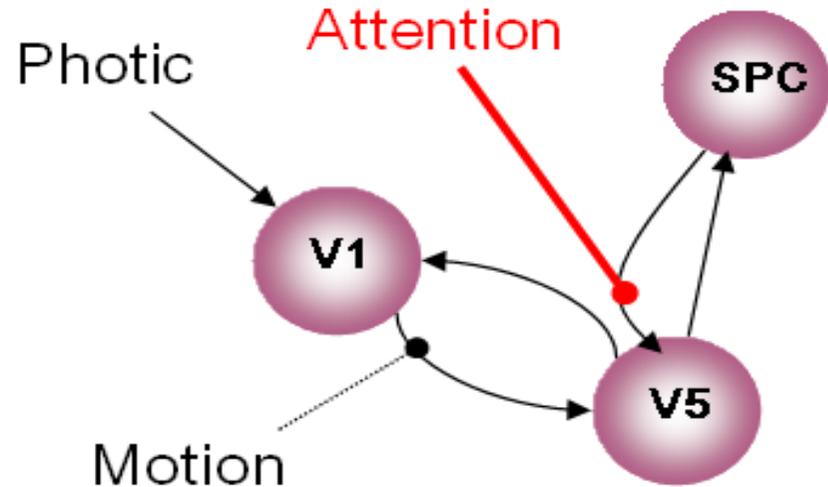


# Effects of attention on motion responses in V5



**Model 1/Hypothesis 1  
Bottom-up Model**

**Model 2/ Hypothesis 2  
Top-down Model**



## Ingredients for a DCM

Specific hypothesis/question

Model: based on hypothesis

Timeseries: from the SPM

Inputs: from design matrix

# Overview

- Dynamic causal models for fMRI
  - Work through example: Attention to Motion
  - Neural level & Hemodynamic level
  - Parameter estimation, priors & inference
- Dynamic causal models for Steady State Responses (rat LFPs)
  - Work through example: Isoflurane effects on connectivity
  - Neural mass model

# Dynamic Causal Models and Physiological Inference: A Validation Study Using Isoflurane Anaesthesia in Rodents

Rosalyn J. Moran<sup>1\*</sup>, Fabienne Jung<sup>3</sup>, Tetsuya Kumagai<sup>3</sup>, Heike Endepols<sup>3</sup>, Rudolf Graf<sup>3</sup>, Raymond J. Dolan<sup>1</sup>, Karl J. Friston<sup>1</sup>, Klaas E. Stephan<sup>1,2</sup>, Marc Tittgemeyer<sup>3</sup>

<sup>1</sup> Wellcome Trust Centre for Neuroimaging, Institute of Neurology, University College London, London, United Kingdom, <sup>2</sup> Laboratory for Social and Neural Systems Research, Department of Economics, University of Zurich, Zurich, Switzerland, <sup>3</sup> Max Planck Institute for Neurological Research, Cologne, Germany

## Abstract

Generative models of neuroimaging and electrophysiological data present new opportunities for accessing hidden or latent brain states. Dynamic causal modeling (DCM) uses Bayesian model inversion and selection to infer the synaptic mechanisms underlying empirically observed brain responses. DCM for electrophysiological data, in particular, aims to estimate the relative strength of synaptic transmission at different cell types and via specific neurotransmitters. Here, we report a DCM validation study concerning inference on excitatory and inhibitory synaptic transmission, using different doses of a volatile anaesthetic agent (isoflurane) to parametrically modify excitatory and inhibitory synaptic processing while recording local field potentials (LFPs) from primary auditory cortex (A1) and the posterior auditory field (PAF) in the auditory belt region in rodents. We test whether DCM can infer, from the LFP measurements, the expected drug-induced changes in synaptic transmission mediated via fast ionotropic receptors; i.e., excitatory (glutamatergic) AMPA and inhibitory GABA<sub>A</sub> receptors. Cross- and auto-spectra from the two regions were used to optimise three DCMs based on biologically plausible neural mass models and specific network architectures. Consistent with known extrinsic connectivity patterns in sensory hierarchies, we found that a model comprising forward connections from A1 to PAF and backward connections from PAF to A1 outperformed a model with forward connections from PAF to A1 and backward connections from A1 to PAF and a model with reciprocal lateral connections. The parameter estimates from the most plausible model indicated that the amplitude of fast glutamatergic excitatory postsynaptic potentials (EPSPs) and inhibitory postsynaptic potentials (IPSPs) behaved as predicted by previous neurophysiological studies. Specifically, with increasing levels of anaesthesia, glutamatergic EPSPs decreased linearly, whereas fast GABAergic IPSPs displayed a nonlinear (saturating) increase. The consistency of our model-based *in vivo* results with experimental *in vitro* results lends further validity to the capacity of DCM to infer on synaptic processes using macroscopic neurophysiological data.

**Citation:** Moran RJ, Jung F, Kumagai T, Endepols H, Graf R, et al. (2011) Dynamic Causal Models and Physiological Inference: A Validation Study Using Isoflurane Anaesthesia in Rodents. *PLoS ONE* 6(8): e22790. doi:10.1371/journal.pone.0022790

**Editor:** Vladimir N. Uversky, University of South Florida College of Medicine, United States of America

**Received** March 1, 2011; **Accepted** July 6, 2011; **Published** August 2, 2011

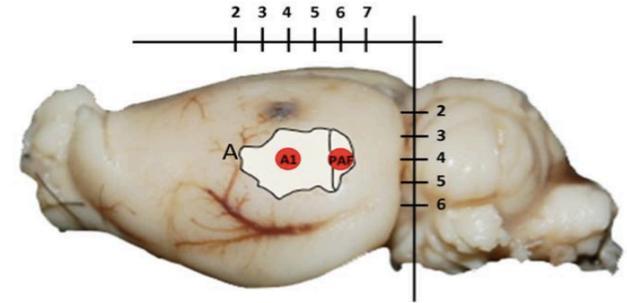
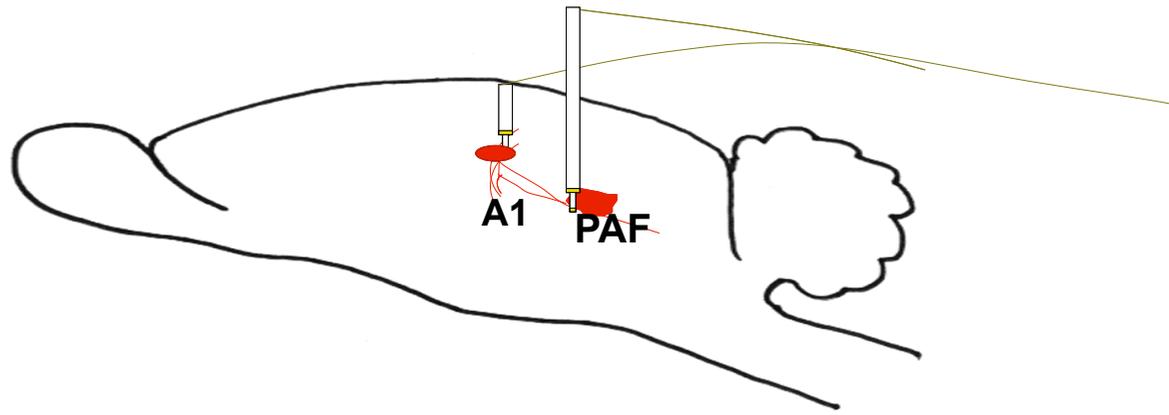
**Copyright:** © 2011 Moran et al. This is an open-access article distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.

**Funding:** This work was supported by the Max Planck Society (MT, FJ, TK, HE, RG, RJM), the NEUROCHOICE project of SystemsX.ch (KES), and the Wellcome Trust (RJD, KJF, RJM). The funders had no role in study design, data collection and analysis, decision to publish, or preparation of the manuscript.

**Competing Interests:** The authors have declared that no competing interests exist.

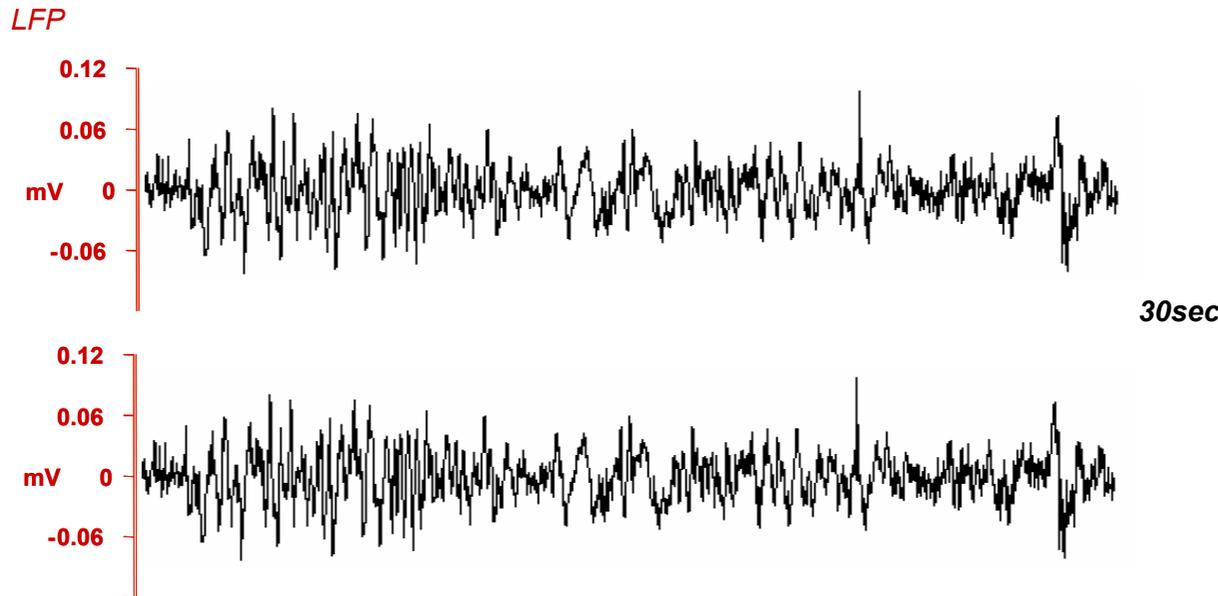
\* E-mail: r.moran@fil.ion.ucl.ac.uk

# Anaesthesia Depth



● = Silverball electrode, diameter: 1 mm

**Figure 1. Electrode Placement.** Electrode placement (silverball electrodes) in primary auditory cortex (A1) and posterior auditory field (PAF) in auditory cortex (A). The anatomical labelling of auditory fields was taken from [38] and matched to a rat brain from our animals. The indicated scaling is in mm.  
doi:10.1371/journal.pone.0022790.g001

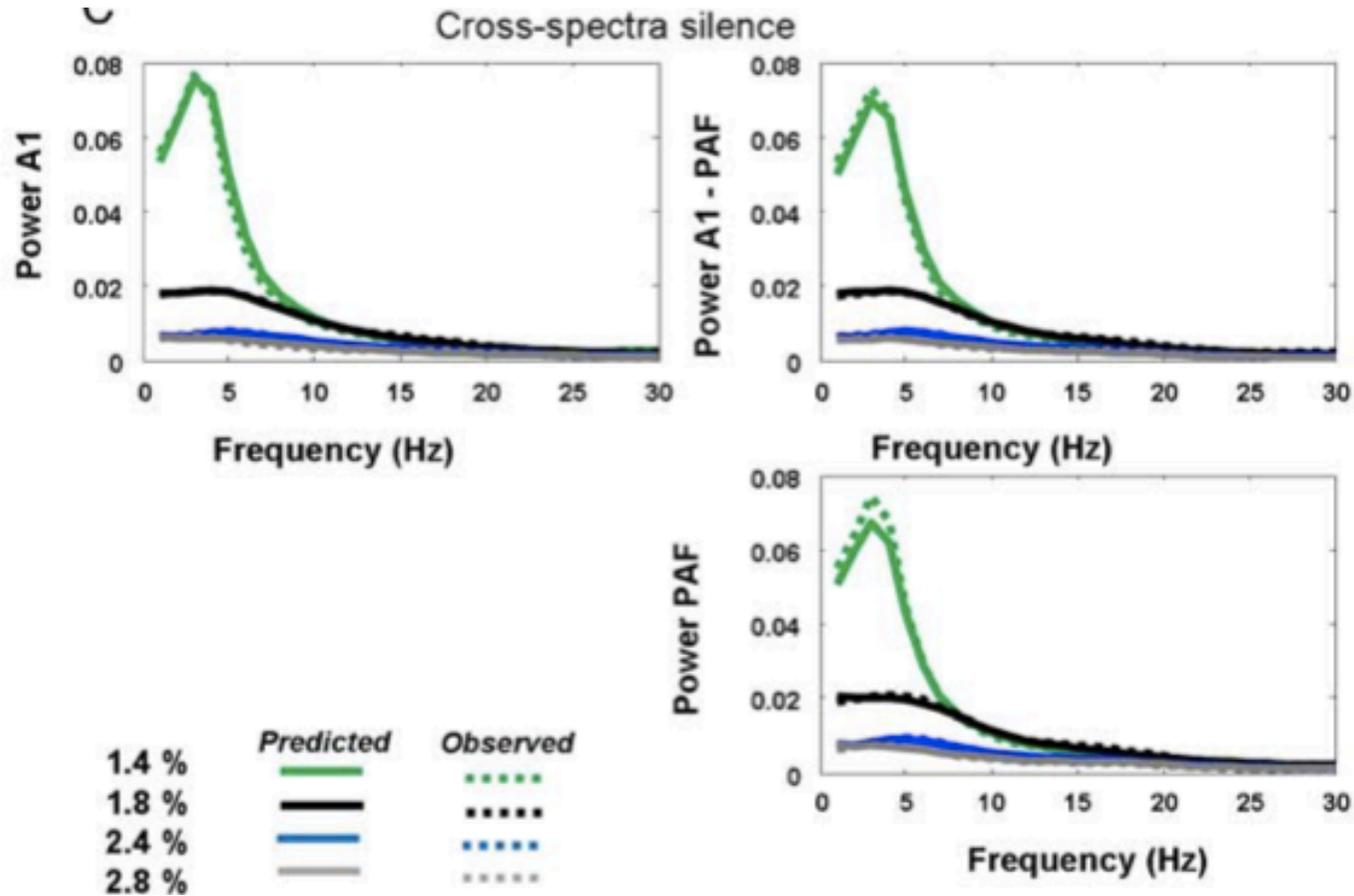


## Trial:

- 1: 1.4 Mg Isoflourane
- 2: 1.8 Mg Isoflourane
- 3: 2.4 Mg Isoflourane
- 4: 2.8 Mg Isoflourane
- 5: awake

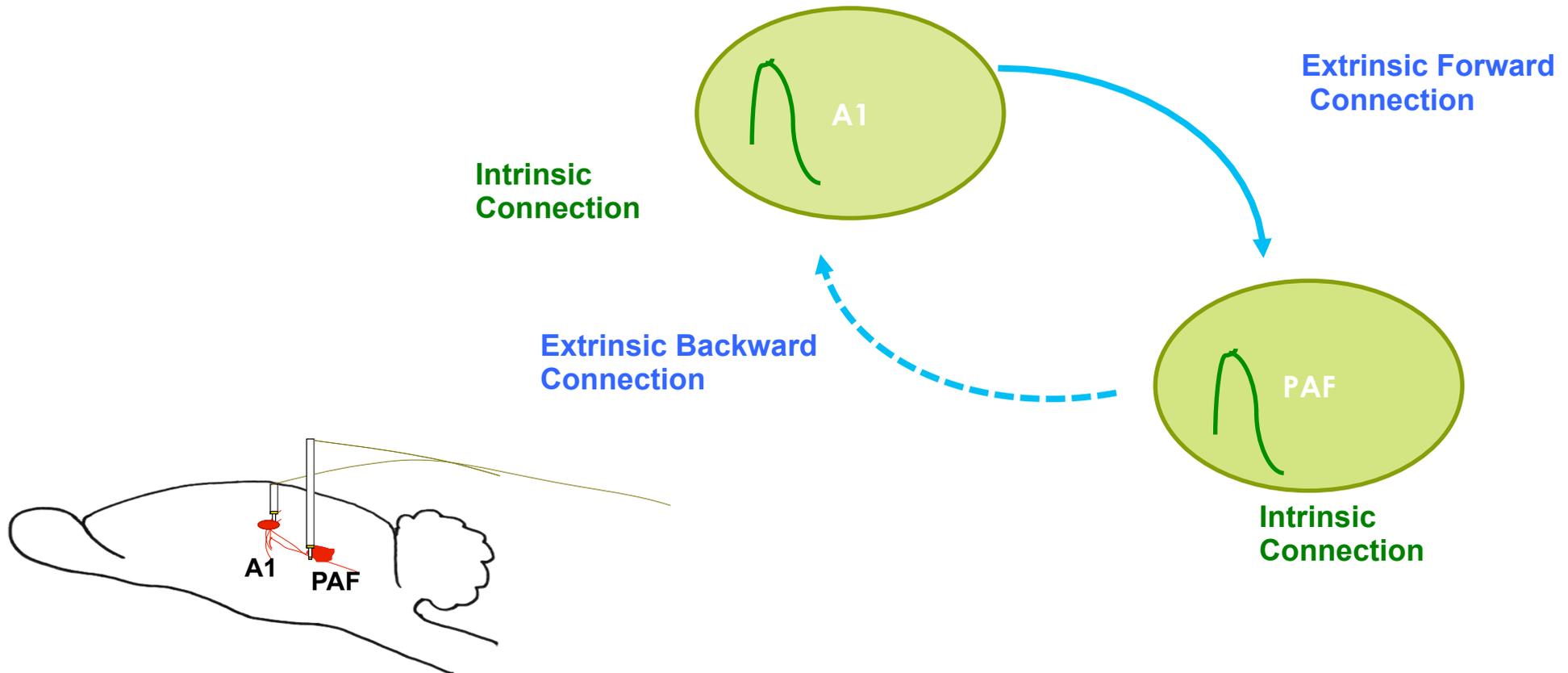
(1 per condition)

# Steady-State Spectral Responses



# Hypothesised mechanisms of action

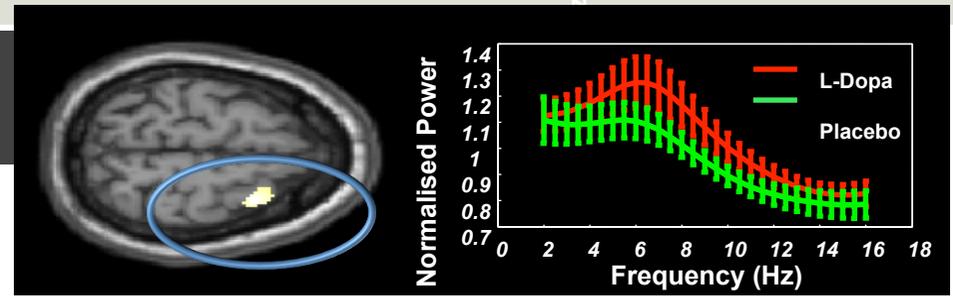
Connectivity effected by Isoflurane:  
Extrinsic or Intrinsic? (Bayesian Model Comparison)  
How so? (Posterior Parameter Estimates)



# Overview

- Dynamic causal models for fMRI
  - Work through example: Attention to Motion
  - Neural level & Hemodynamic level
  - Parameter estimation, priors & inference
  
- Dynamic causal models for Steady State Responses (rat LFPs)
  - Work through example: Isoflurane effects on connectivity
  - Neural mass model

# Neural Mass Model



Lateral connections

Intrinsic connections

$\gamma_5$

Backward connections

input

$u$

Forward connections

*Inhibitory cells in supragranular layers*

$$\dot{x}_7 = x_8$$

$$\dot{x}_8 = \kappa_e H_e (B + \gamma_3 I) S(x_9) - 2\kappa_e x_8 - \kappa_e^2 x_7$$

$$\dot{x}_{10} = x_{11}$$

$$\dot{x}_{11} = \kappa_i H_i \gamma_5 S(x_{12}) - 2\kappa_i x_{11} - \kappa_i^2 x_{10}$$

$$\dot{x}_{12} = x_8 - x_{11}$$

$\gamma_4$

$\gamma_3$

*Excitatory spiny cells in granular layers*

$$\dot{x}_1 = x_4$$

$$\dot{x}_4 = \kappa_e H_e ((F + \gamma_1 I) S(x_9) + Cu) - 2\kappa_e x_4 - \kappa_e^2 x_1$$

$\gamma_1$

$\gamma_2$

*Excitatory pyramidal cells in infragranular layers*

$$\dot{x}_2 = x_5$$

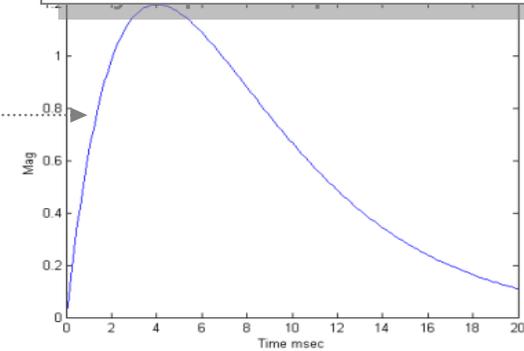
$$\dot{x}_5 = \kappa_e H_e (BS(x_9) + \gamma_2 S(x_1)) - 2\kappa_e x_5 - \kappa_e^2 x_2$$

$$\dot{\mu}_3 = x_6$$

$$\dot{x}_6 = \kappa_i H_i \gamma_4 S(x_{12}) - 2\kappa_i x_6 - \kappa_i^2 x_3$$

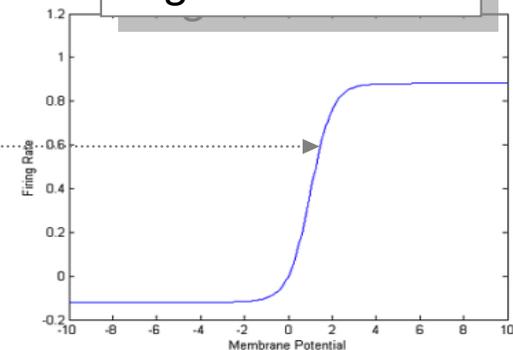
$$\dot{x}_9 = x_5 - x_6$$

Synaptic 'alpha' kernel



$$\dot{x} = f(x, u)$$

Sigmoid function



# Time to Frequency Domain

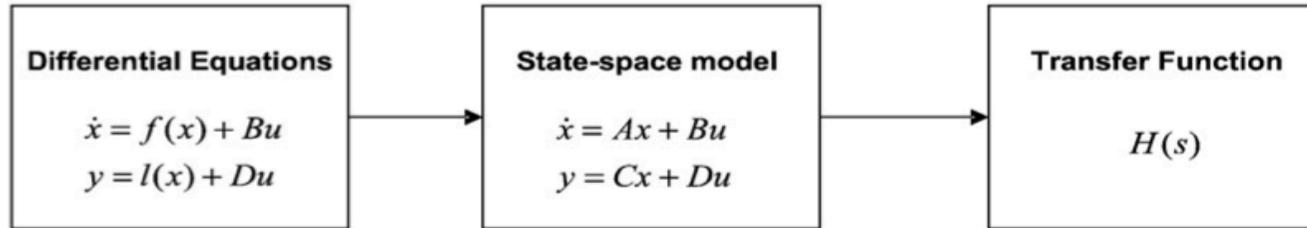


Fig. 1. Conversion scheme to obtain spectral outputs from the systems transfer function.

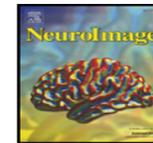
NeuroImage 44 (2009) 796–811



Contents lists available at [ScienceDirect](http://www.sciencedirect.com)

NeuroImage

journal homepage: [www.elsevier.com/locate/ynimg](http://www.elsevier.com/locate/ynimg)



DCM for SSR

## Dynamic causal models of

R.J. Moran <sup>a,\*</sup>, K.E. Stephan <sup>a,b</sup>, T. ...

<sup>a</sup> Wellcome Trust Centre for Neuroimaging, Institute of N  
<sup>b</sup> Laboratory for Social and Neural Systems Research, Ins  
<sup>c</sup> Institute of Physiology, University of Münster, Germany

DCM for CSD

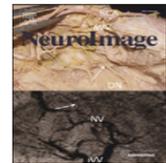
NeuroImage 59 (2012) 439–455



Contents lists available at [ScienceDirect](http://www.sciencedirect.com)

NeuroImage

journal homepage: [www.elsevier.com/locate/ynimg](http://www.elsevier.com/locate/ynimg)



Technical Note

## DCM for complex-valued data: Cross-spectra, coherence and phase-delays

K.J. Friston <sup>a</sup>, A. Bastos <sup>b,c</sup>, V. Litvak <sup>a</sup>, K.E. Stephan <sup>a,d</sup>, P. Fries <sup>c</sup>, R.J. Moran <sup>a,\*</sup>

<sup>a</sup> The Wellcome Trust Centre for Neuroimaging, University College London, Queen Square, London WC1N 3BG, UK

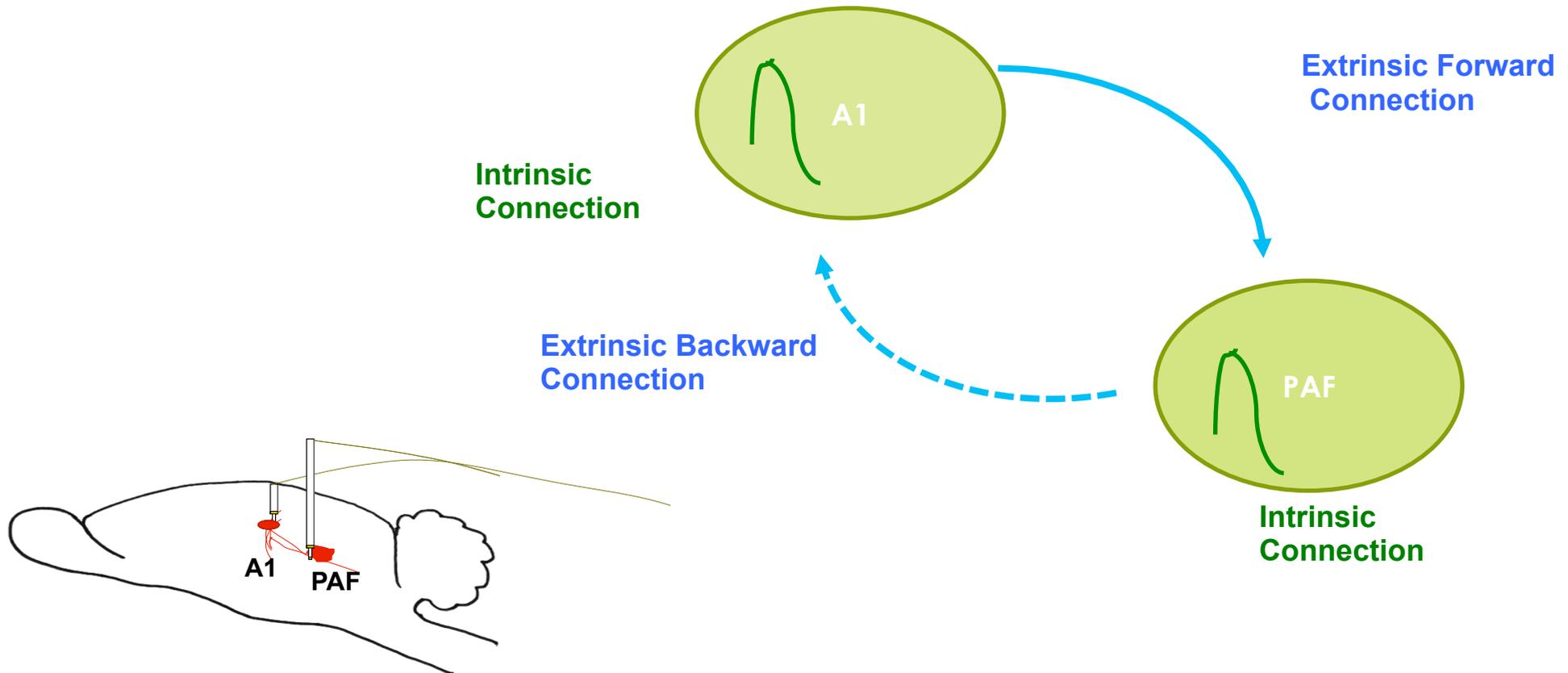
<sup>b</sup> Center for Neuroscience and Center for Mind and Brain, University of California-Davis, Davis, CA 95618, USA

<sup>c</sup> Ernst Strüngmann Institute in Cooperation with Max Planck Society, Deutschordenstraße 46, 60528 Frankfurt, Germany

<sup>d</sup> Laboratory for Social and Neural Systems Research, Dept. of Economics, University of Zurich, Switzerland

# Hypothesised mechanisms of action

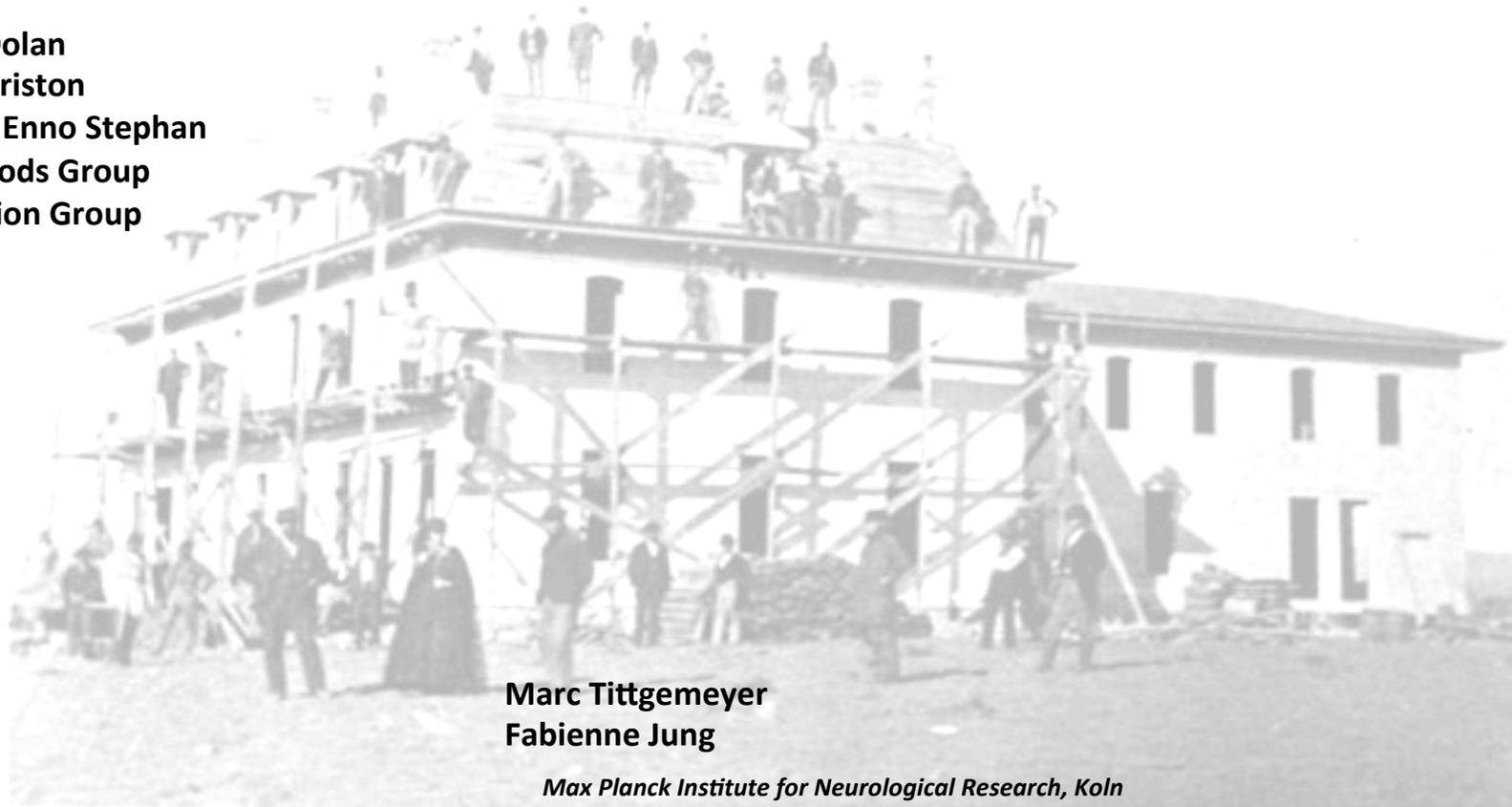
Connectivity effected by Isoflurane:  
Extrinsic or Intrinsic? (Bayesian Model Comparison)  
How so? (Posterior Parameter Estimates)



# Thank You

## Acknowledgments

Ray Dolan  
Karl Friston  
Klaas Enno Stephan  
Methods Group  
Emotion Group



Marc Tittgemeyer  
Fabienne Jung

*Max Planck Institute for Neurological Research, Koln*