

Continuous Time Modeling:

On the role of time in the search for mechanisms of the human mind, brain, and behavior

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An Example: The Case of Dr. M.



The Case of Dr. M. Act 1: Consulting the Literature

Between-Person (BP) correlation:



The Case of Dr. M. Act 1: Consulting the Literature



Dr. M.

The Case of Dr. M. Act 2: Running a WP Study

Within-Person (WP) correlation (T = 210 days):



The Case of Dr. M. Act 2: Running a WP Study

Within-Person (WP) correlation (T = 210 days):



6

The Case of Dr. M. Act 2: Running a WP Study



Dr. M.



Dr. M.

Dr. X.

Dynamic Within-Person (WP) Models:



 $\Delta_t = 24$ hours



Dynamic Within-Person (WP) Models:







Dr. M.

Dr. X.



Interventions:



Interventions and Time:





The end

The Case of Dr. M.: A Summary

- BP and WP effects, obtained from repeatedly assessing the same individual over <u>time</u> need not be the same.
- While WP research eliminates the problem of heterogeneity across individuals, it introduces the problem of heterogeneity in <u>time</u>.
- While dynamic (lead-lag) models help in the identification of causal mechanisms, results are difficult to interpret and compare without explicit reference to <u>time</u>.
- (WP) effects due to interventions are difficult to interpret and compare without considering the <u>time</u> course.

Part I. Another dimension, another scientific paradigm?

Part II. Another dimension or another variable?



Part III. Another dimension, many complexities?

The Goal

... is to understand the mechanisms of the human mind, brain, and behavior.



➢ Following Pearl (2009), I will define a causal effect as:

P(Y = y | do(x))

> Via
$$E(Y|do(x')) - E(Y|do(x''))$$

this may be linked to Rubin's model for causal inference.

While the link to inference is obvious, note the difference in the definition

$$Y_{i|d=1} - Y_{i|d=0}$$

However, in this presentation a pragmatic approach to causality is taken, with potential differences between theoretical approaches being irrelevant to all main arguments.

(Pearl, 2009; Holland, 1986, Halaby, 2004)

Another dimension, another scientific paradigm?

- 1. The statistical perspective:
 - > Usually $Ø_{BP} ≠ Ø_{WP}$

 $\bigwedge^{}_{BP} = +0.3 \text{ vs. } \phi_{WP} = -0.05$

Usually BP models will differ from WP models:



- **1.** The statistical perspective:
 - In case of a saturated model, unconditional structural equivalence ("ergodicity") is given if

$$\mathbf{\Theta}_i = \mathbf{\Theta}_t$$

for all possible combinations of *t* and *i* (e.g., with $\theta_i = {\mu_i, \Sigma_i}$ and $\theta_t = {\mu_t, \Sigma_t}$)

1. The statistical perspective:

"only under very strict conditions—which are hardly obtained in real psychological processes—can a generalization be made from a structure of interindividual variation to the analogous structure of intraindividual variation". (Molenaar, 2004, p. 201)

The New Person-Specific Paradigm in Psychology

Peter C.M. Molenaar and Cynthia G. Campbell

1. Another dimension, another scientific paradigm?

- 2. The causal perspective:
 - > What *caused* the data?
 - Common causes (what leads to "ergodicity")?

$$P_{\theta_i}(Y = y | do(x)) = P_{\theta_t}(Y = y | do(x))$$



Unique causes (what destroys "ergodicity")?



1. Another dimension, another scientific paradigm?

3. The concept of conditional (BP/WP) equivalence:



1. Another dimension, another scientific paradigm?

3. The concept of conditional (BP/WP) equivalence:



Voelkle, Brose, Schmiedek, & Lindenberger (2014). *Multivariate Behavioral Research.* Voelkle, Oud, von Oertzen, & Lindenberger (2012). *Structural Equation Modeling.*

Another Dimension or Another Variable?





"...although time is inextricably linked to the concept of development, in itself it cannot explain any aspect of developmental change....

Time, rather like the theatrical stage upon which the processes of development are played out, provides a necessary base upon which the description, explanation, and modification of development proceed."

(Baltes et al. 1988, p. 108)

2.

Another Dimension or Another Variable?



Another Dimension or Another Variable?

Researcher 1:



Researcher 2:



2. Another Dimension or Another Variable?

(Latent) Cross-Lagged Panel Designs With Different Intervals:



> The Problem:

$$\eta_u = A\eta_{u-1} + b_u + Mx_u + \zeta_u$$

> The Solution:

$d\mathbf{\eta}(t) = (\mathbf{A}\mathbf{\eta}(t) + \mathbf{b} + \mathbf{M}\mathbf{x}(t))dt + \mathbf{G}d\mathbf{W}(t)$

Continuous Time Models

2. A Super-Quick Introduction to CT Models



Voelkle, Oud, Davidov, & Schmidt (2012). Psychological Methods.

2. A Super-Quick Introduction to CT Models

1. The complete discrete time model:

$$\mathbf{x}(t_i) = \mathbf{A}(\Delta t_i)\mathbf{x}(t_i - \Delta t_i) + \mathbf{b}(\Delta t_i) + \mathbf{w}(\Delta t_i)$$

2. Take the derivative with respect to time:

$$\frac{\mathrm{d}\mathbf{x}(t)}{\mathrm{d}t} = \mathbf{A}\,\mathbf{x}(t) + \mathbf{b} + \mathbf{G}\frac{\mathrm{d}\mathbf{W}(t)}{\mathrm{d}t}$$

3. Solve (2) for initial time point and given time interval:

$$\mathbf{x}(t) = e^{\mathbf{A} \cdot (t-t_0)} \mathbf{x}(t_0) + \mathbf{A}^{-1} \left[e^{\mathbf{A} \cdot (t-t_0)} - \mathbf{I} \right] \mathbf{b} + \int_{t_0}^{t} e^{\mathbf{A} \cdot (t-s)} \mathbf{G} d\mathbf{W}(s)$$

with $cov \left[\int_{t_0}^{t} e^{\mathbf{A} \cdot (t-s)} \mathbf{G} d\mathbf{W}(s) \right] = \int_{t_0}^{t} e^{\mathbf{A} \cdot (t-s)} \mathbf{Q} e^{\mathbf{A}' \cdot (t-s)} ds = irow \{ \mathbf{A}_{\#}^{-1} \left[e^{\mathbf{A}_{\#} \cdot (t-t_0)} - \mathbf{I} \right] row \mathbf{Q} \}$
for $\mathbf{Q} = \mathbf{G}\mathbf{G}'$ and $\mathbf{A}_{\#} = \mathbf{A} \otimes \mathbf{I} + \mathbf{I} \otimes \mathbf{A}$

4. Impose constraints between discrete and continuous time parameters

$$\mathbf{x}(t_i) = \mathbf{e}^{\mathbf{A} \cdot \Delta t_i} \mathbf{x}(t_i - \Delta t_i) + \mathbf{A}^{-1} [\mathbf{e}^{\mathbf{A} \cdot \Delta t_i} - \mathbf{I}] \mathbf{b} + \mathbf{w}(\Delta t_i)$$

5. Put (e.g.) into SEM format with:

 $\mathbf{y} = \mathbf{\Lambda} \mathbf{\eta} + \mathbf{\epsilon}$ with $cov(\mathbf{\epsilon}) = \mathbf{\Theta}$ representing the usual measurement equation, and $\eta = B\eta + \zeta$ with $cov(\zeta) = \Psi$. with $\mathbf{\eta}' = (\mathbf{x}(t_0)' \quad \mathbf{x}(t_1)' \quad \mathbf{x}(t_i)' \quad \cdots \quad \mathbf{x}(t_{i=T})' \quad 1)'$ $\boldsymbol{\zeta}' = ([\boldsymbol{x}(t_0) - \boldsymbol{\mu}_{\boldsymbol{x}(t_0)}]' \quad [\boldsymbol{w}(t_1 - \Delta t_1)]' \quad [\boldsymbol{w}(t_i - \Delta t_i)]' \quad \cdots \quad [\boldsymbol{w}(t_{i=T} - \Delta t_{i=T})]' \quad 1)'$ $\mathbf{B} = \begin{pmatrix} \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} & \mathbf{\mu}_{\mathbf{x}(t_0)} \\ e^{\mathbf{A} \cdot \Delta t_1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}^{-1}[e^{\mathbf{A} \cdot \Delta t_1} - \mathbf{I}]\mathbf{b} \\ 0 & e^{\mathbf{A} \cdot \Delta t_i} & \mathbf{0} & \mathbf{0} & \mathbf{A}^{-1}[e^{\mathbf{A} \cdot \Delta t_i} - \mathbf{I}]\mathbf{b} \\ \vdots & \ddots & & & \\ \mathbf{0} & \mathbf{0} & e^{\mathbf{A} \cdot \Delta t_{i=T}} & \mathbf{0} & \mathbf{A}^{-1}[e^{\mathbf{A} \cdot \Delta t_{i=T}} - \mathbf{I}]\mathbf{b} \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \quad \mathbf{\Psi} = \begin{pmatrix} \mathbf{\Psi}_{\mathbf{x}(t_0)} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_{\Delta t_1} & & & \\ \vdots & & \ddots & & \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{Q}_{\Delta t_i=T} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{Q}_{\Delta t_{i=T}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix}$

6. Estimate parameters (e.g. via ML):

$$LL = \frac{-Nm}{2}\log(2\pi) - \frac{N}{2}\log|\boldsymbol{\Sigma}| - \frac{1}{2}\sum_{i=1}^{N}(\boldsymbol{y}_i - \boldsymbol{\mu})' \boldsymbol{\Sigma}^{-1}(\boldsymbol{y}_i - \boldsymbol{\mu})$$

36

2. A Super-Quick Introduction to CT Models



2. A Super-Quick Introduction to CT Models

To deal with the math behind continuous time modeling, we developed *ctsem*:





Output: Continuous time parameter estimates & standard errors

-2 log likelihood:

fr	ee para	ameters	:				
	name	matrix	row	col	Estimate	Std	.Error
1	phi11	S	1	1	0.632860723	0.017	155319
2	phi21	S	2	1	0.244730438	0.011	330932
3	phi22	S	2	2	0.457576808	0.012	403227
4	F11	DRIFT	1	1	-0.447282043	0.019	877226
5	F21	DRIFT	2	1	0.043292845	0.009	631034
6	F12	DRIFT	1	2	0.232497994	0.018	058916
7	F22	DRIFT	2	2	-0.117466337	0.008	651670
8	cint1	CINT	1	1	0.536314454	0.046	290824
9	cint2	CINT	1	2	0.220003765	0.022	199724
10	q11	Q	1	1	0.473242113	0.015	697220
11	q12	Q	2	1	-0.004610147	0.005	495074
12	q22	Q	2	2	0.154509580	0.003	846262
13	m1	М	1	F1	2.503428393	0.015	249241
14	m2	М	1	F2	2.842722000	0.012	965461
observed statistics: 13375							
estimated parameters: 14							
degrees of freedom: 13361							Drift matrix

23415.93

Plots: Autoregressive and cross-lagged parameters



Continuous time models resolve the problem of unequal time intervals,

and....

...permit the analysis of panel models, N = 1 time series (dynamic factor models), and the integration of BP and WP models



...comprise latent change scores models as a special case



Voelkle & Oud (2015). Structural Equation Modeling.

...permit the estimation of oscillating and/or higher order (coupled) processes



...may help in improving study designs by optimizing sampling and identifying "optimal lags" (Dormann, 2015, p. 597)



Time

...may help in improving study designs by optimizing sampling and identifying "optimal lags"



...may help in improving study designs by optimizing sampling and identifying "optimal lags"



...offers new perspectives on missing values (which "do not exist") in continuous time



...permits various extensions, including time-varying and timeinvariant predictors and random effects ("traits)



$d\mathbf{\eta}(t) = (\mathbf{A}\mathbf{\eta}(t) + \mathbf{b} + \mathbf{M}\mathbf{x}(t))dt + \mathbf{G}d\mathbf{W}(t)$

... is currently being extended to a fully hierarchical Bayesian approach by means of rstan



...is related to Gaussian Process Panel (GPPM) models, which provides additional flexibility, e.g. in the choice of kernel function and person specific predictions





Another Dimension, Many Complexities?

3. Another Dimension, Many Complexities?

- Yes...as we get more familiar with the unknown third dimension, the loose ends become obvious.
- We may distinguish between <u>statistical</u> and <u>conceptual</u> problems and opportunities.



1. Statistical problems and opportunities:

A) Computational limitations with respect to the number of individuals, time points, and variables/processes.



1. Statistical problems and opportunities:

B) Better approaches to control or observed and unobserved heterogeneity across individuals and time



2. Conceptual problems and opportunities:

A) We need to reconsider traditional approaches to mediation analysis from a continuous time perspective



2. Conceptual problems and opportunities:

B) Need to better connect continuous time approaches to established frameworks of causal inference (e.g., DAGs)



So...where to go from here? An Outlook

An Outlook

...on (the future of) studying the mechanisms of the human, mind, brain, and behavior



1. Cast theories in terms of dynamic models:



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d\mathbf{\eta}(t) = (\mathbf{A}\mathbf{\eta}(t) + \mathbf{b} + \mathbf{M}\mathbf{x}(t))dt + \mathbf{G}d\mathbf{W}(t)
```



- = "thinking healthy" (cognition)
- = "being happy" (affective experience)
 - = "eating unhealthy" (behavior)

3.

2. Optimize the (sampling) design to yield maximum information about the presumed dynamic mechanisms:



3. Estimate (and understand!) the underlying continuous time dynamics:



4. Study the causal nature of the system:



5. Develop targeted interventions <u>based</u> on the entire system:



6. Develop targeted interventions to change the system:



3.

An Outlook

7. <u>Anticipate</u> the effect of targeted interventions based on an <u>altered</u> system:



Summary

Part I. Another dimension, another scientific paradigm?

maybe...

Part II. Another dimension or another variable?

it depends...

Part III. Another dimension, many complexities?

yes!

The Goal

...is to understand the mechanisms of the human mind, brain, and behavior.



Thank you very much for your attention!

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